

# PREFACE

## *SEASON'S GREETINGS TO ALL*

I am very much delighted to present this book **ENGINEERING MATHEMATICS-IV** composed according to the latest VTU syllabus for the IV semester B.E degree course. It also caters to the need of autonomous engineering institutions in Karnataka and other technological universities in the country.

This **student friendly text book** is equipped with all the salient features of my earlier semester books that has overwhelming response from the community of both students and teachers. This is the source of inspiration for the author and publishers.

Two of my senior colleagues in the department **Dr. D. Mamta** and **Smt. G. V. Pankaja** have meticulously scrutinized the content of the book in an exemplary manner and I am highly indebted to them.

I wholeheartedly thank **Mr. K. V. Balakrishna** of M/s Sudha Publications for publishing this book with quality features.

I very much appreciate and thank for the systematic computer type setting work executed by **Mr. S. Raghunandhan** and team of 'Allkind'. Also I thank the printers for the elegant offset printing.

Constructive suggestions from all ends will be accepted with humility.

Finally, it is my privilege to wish every one '**A HAPPY & PROSPEROUS NEW YEAR - 2012**'

January 24th, 2012  
Maagha Shukla Paadya  
Mysore - 8

**K.S.Chandrashekar**

## REWARD

*VTU students of the current scheme w.e.f 2010-11, scoring 125/125 in all four papers of Engineering Mathematics I to IV Semesters (10 MAT 11, 21, 31, 41) will be rewarded with a cash prize of Rs.10,000/- by the author. Please write to the author directly along with attested xerox copies of marks cards of all the four semesters.*

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**Achiever : Mr. Bharath M.V, a student from 2006 batch of E&C branch from PESIT, Bangalore, received cash prize during 2008.**

## **SYLLABUS ENGINEERING MATHEMATICS - IV**

**Code : 10 MAT 41**

**Hrs / week : 04**

**Total Hrs : 52**

**IA Marks : 25**

**Exam Hrs : 03**

**Exam Marks : 100**

### **PART - A Unit - I : NUMERICAL METHODS - 1**

Numerical solution of ordinary differential equations of first order and first degree; Picard's method, Taylor's series method, modified Euler's method; Runge-Kutta method of fourth-order. Milne's and Adams - Bashforth predictor and corrector methods (No derivations of formulae) **[6 hours]**

#### **Unit - II : NUMERICAL METHODS - 2**

Numerical solution of simultaneous first order ordinary differential equations: Picard's method, Runge-Kutta method of fourth-order. Numerical solution of second order ordinary differential equations: Picard's method, Runge-Kutta method and Milne's method.

**[6 hours]**

#### **Unit - III : COMPLEX VARIABLES - 1**

Function of a complex variable, Analytic functions, Cauchy-Riemann equations in cartesian and polar forms. Properties of analytic functions. Application to flow problems - complex potential, velocity potential, equipotential lines, stream functions, stream lines. **[7 hours]**

#### **Unit - IV : COMPLEX VARIABLES - 2**

Conformal transformations : Bilinear transformations. Discussion of transformations :  $w = z^2$ ,  $w = e^z$ ,  $w = z + (a^2/z)$ . Complex line integrals- Cauchy's theorem and Cauchy's integral formula. **[7 hours]**

### **PART - B Unit - V : SPECIAL FUNCTIONS**

Solution of Laplace equation in cylindrical and spherical systems leading Bessel's and Legendre's differential equations. Series solution of Bessel's differential equation leading to Bessel function of first kind, orthogonal property of Bessel functions.

Series solution of Legendre's differential equation leading to Legendre polynomials, Rodrigue's formula. **[7 hours]**

#### **Unit - VI : PROBABILITY THEORY - 1**

Probability of an event, empirical and axiomatic definition, probability associated with set theory, addition law, conditional probability, multiplication law, Baye's theorem. **[6 hours]**

#### **Unit - VII : PROBABILITY THEORY - 2**

Random variables (discrete and continuous), probability density function, cumulative density function. Probability distributions - Binomial and Poisson distributions; Exponential and normal distributions. **[7 hours]**

#### **Unit - VIII : SAMPLING THEORY**

Sampling, sampling distributions, standard error, test of hypothesis for means, confidence limits for means, Student's *t* distribution. Chi-square distribution as a test of goodness of fit.

**[6 hours]**

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